Mean flow generation by geometrically focused gyroscopic waves

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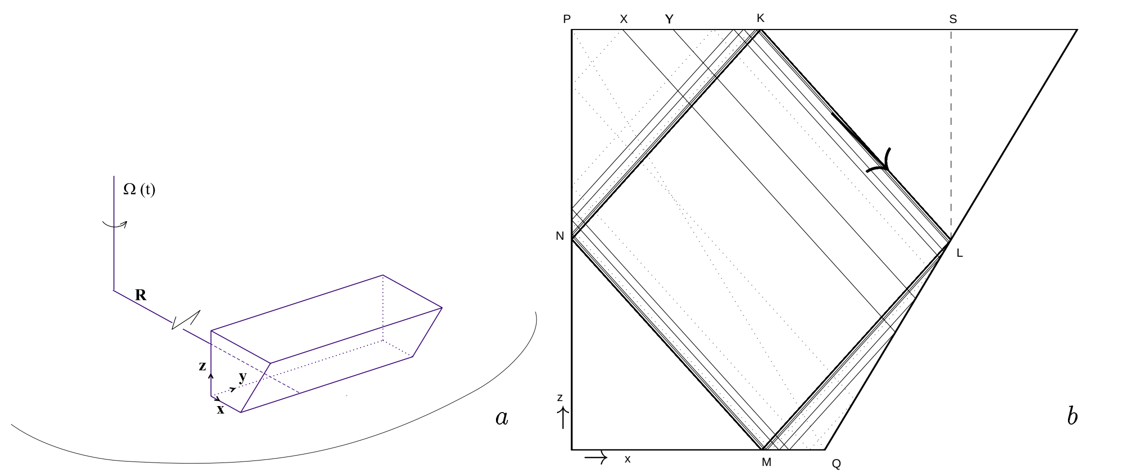
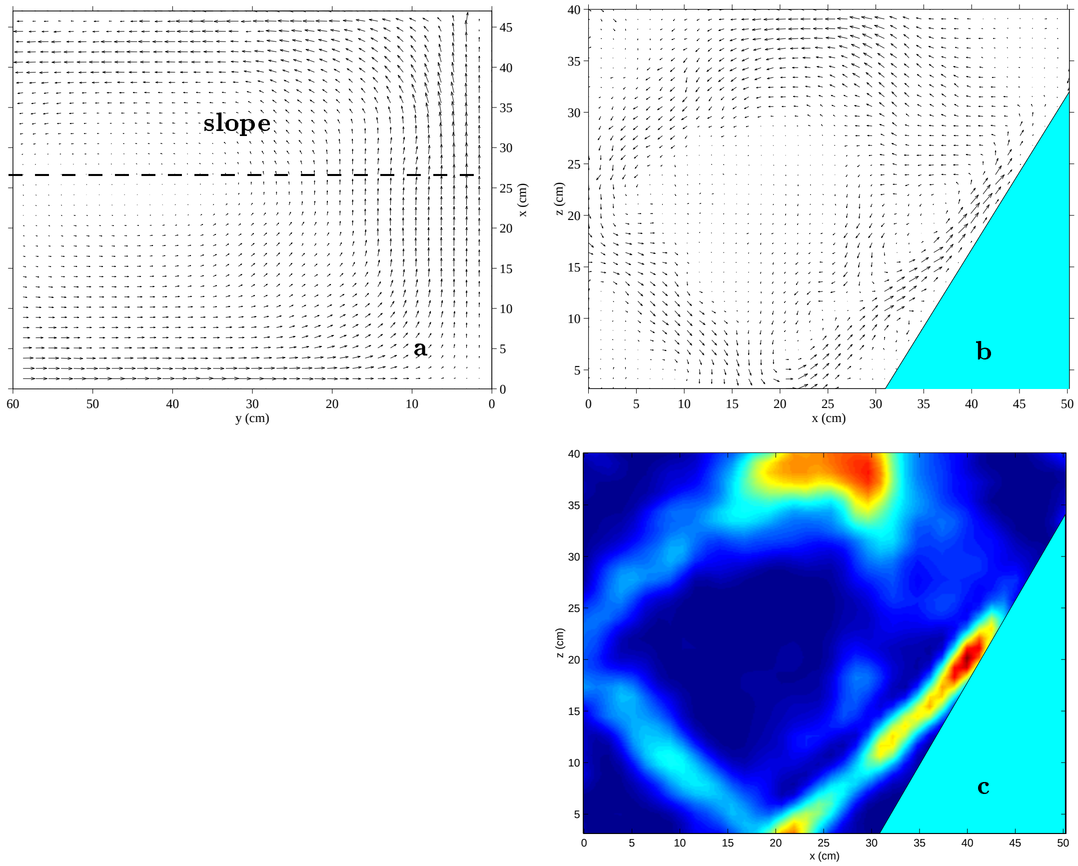
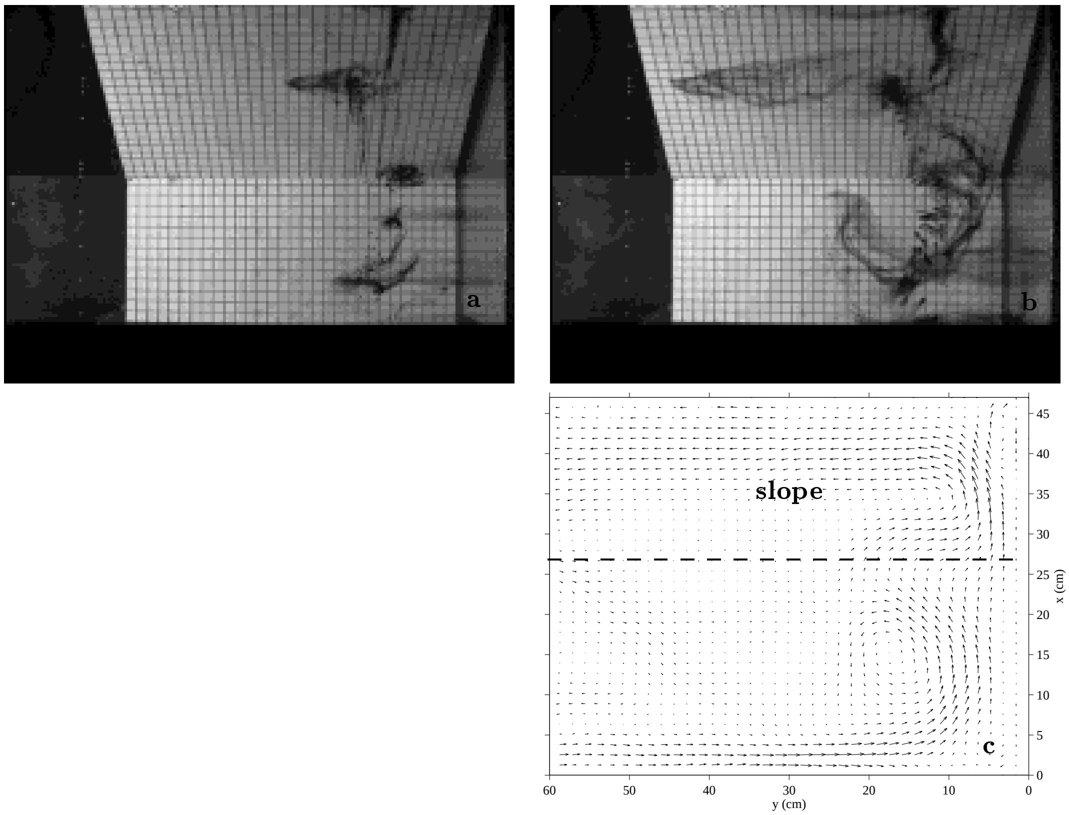
A homogeneous fluid in stationary rotation is stably-stratified in angular momentum (increasing radially). When this dynamic equilibrium is periodically perturbed, gyroscopic (or inertial) waves result, that propagate through the fluid along an angle with the vertical determined by the ratio of perturbation frequency to twice the angular frequency. Due to this constraint, in an infinitely long channel, such waves are, upon reflecting from sloping side walls, subject to focusing, much like internal waves are. In a tank of finite azimuthal extent, this result can only be approximately true, as the circular current patterns, associated with the gyroscopic wave, need to accomodate the presence of the plane side walls. As long as the tank is much longer in along-slope than cross-slope direction, attractors might still arise, a suggestion confirmed by a laboratory experiment in which the perturbation is generated by modulating the angular velocity of the tank. Focusing gyroscopic waves lead to mixing of angular momentum at the location where the attractor reflects from the sloping wall. This predicts a mean flow above that location, confirmed by velocity measurements and dye spreading.



Figuur 1: (a) Sketch of the tank as excentrically situated on the platform whose rotation speed $\Omega(t) = \Omega_0(1+\epsilon\cos\sigma t)$ is weakly modulated sinusoidally. Distance of the central axis of the tank to the platform's rotation axis is $R = 5.5$ m. The tank is relatively long in its along-slope y-direction (120 cm), compared to its width in x-direction (54 cm) and height in z-direction (40 cm), so that variations in y-direction are probably weak ($\partial_y \approx 0$), rendering predictions based on characteristics in an $x-z$ plane, as in (b), useful.
(b) Sketch of vertical cross-section of container with sloping side wall. The tank is filled with homogeneous fluid, that is 'stratified' in angular momentum (in this 2D-plane, increasing, with increasing x). Some characteristics (lines along which energy propagates) of strictly 2D inertial waves are followed to the right as solid lines (from X and Y), showing their convergence upon reflection from the slope, and to the left as dotted line (from X). For the particular ratio of perturbation and angular frequencies ($T_0 = 2\pi/\Omega_0 = 46.17$ s, $T = 2\pi/\sigma = 32.4$ s, $\epsilon = 0.1$), shown here, these characteristics converge on the square-shaped attractor $KLMN$ (along which energy propagates in clockwise direction). Focusing takes place upon downward reflection at the slope. Point L, is therefore the location where intensification, and hence mixing, occurs first. The mixed fluid will spread out vertically, along LS, the axial centrifugal-potential surface. A second modulation frequency employed in the experiments, corresponds with a degenerate line-attractor, dotted line PQ.



Figuur 2: (a) Current observations (u, v) in horizontal x, y-section of right half of the tank, at $z = 30$ cm above the bottom. The sloping wall is in the upper half of this figure, above the dashed line ($x > 27$ cm). Vertical, solid walls are at $x, y = 0$. The right half of the horizontal, inviscid, alternating vorticity-conserving circulation pattern is visible in this plane. Motions are visualised by illuminating particles with a laser sheet, that are followed by a digital video camera (Particle Image Velocimetry[29]). The laser is kept out-of-focus (1 cm 'sheet width'), enabling tracking of particles despite some cross-sheet displacement. Velocity fields in the plane of the laser sheet are obtained by comparing patterns in two subsequent images (taken 0.25 s apart). The maximum current in this figure is 0.48 cm s^{-1}.
(b) Current observations (u, w) in a vertical x, z-section at $y = 25$ cm from the front wall, showing focused inertial waves of modulation period. The maximum current in this plane is 0.32 cm s^{-1}. Notice that the bottom 3 cm and upper right hand corner have no observations.
(c) False-colour (and smoothed) pattern of in-plane component of kinetic energy, $u^2 + w^2$, in the vertical section displayed in (b) at a particular instant of the quasi-periodic state, revealing the (nearly) square pattern, which the adopted frequency ratio aims at. This in-plane kinetic energy is not by itself conserved. Hence, each of the branches of the attractor vary in intensity, but, overall, its square-shaped pattern remains visible over a cycle.



Figuur 3: (a) Top view of right half of container, with sloping wall visible in upper half of figure. Bottom and sloping wall have been supplemented with a 2 cm grid (distorted along the slope). Dye has been injected through a number of holes (of diameter ≈ 1 mm) in the glass top lid, at 14 cm from the vertical glass wall, at the right ($y = 0$). Modulation period is 32.4 s. Dye pictures, shown here, are taken at the moment that the dye reaches its most rightward position over the slope.
(b) Picture taken 6 modulation periods after that in (a), showing the clear leftward spreading of dye over the central part of the slope in the upper half of the figure (revealing a cyclonic mean flow, i.e. in the same direction as the anti-clockwise background rotation).
(c) Top view of observed (time-averaged) mean flow at height $z = 30$ cm. The foot of the slope (at $x = 27$ cm) is denoted by a dashed line. Notice the presence of a cyclonic mean flow over the slope (in the upper half of this picture, at $x \approx 40$ cm), which is clearly detached from the intersection of this plane with the slope (at $x = 46$ cm). A return-flow due to continuity (at $x \approx 2$ cm), visible at the bottom of this picture, does stick to the (vertical) wall. The maximum current in this plane is 0.13 cm s^{-1}, which agrees well with the 26 cm displacement of the dye tip, occurring at the mid-slope position over 194.4 s period, inferred from combining panels (a) and (b).