On gravity currents propagating in a stratified ambient

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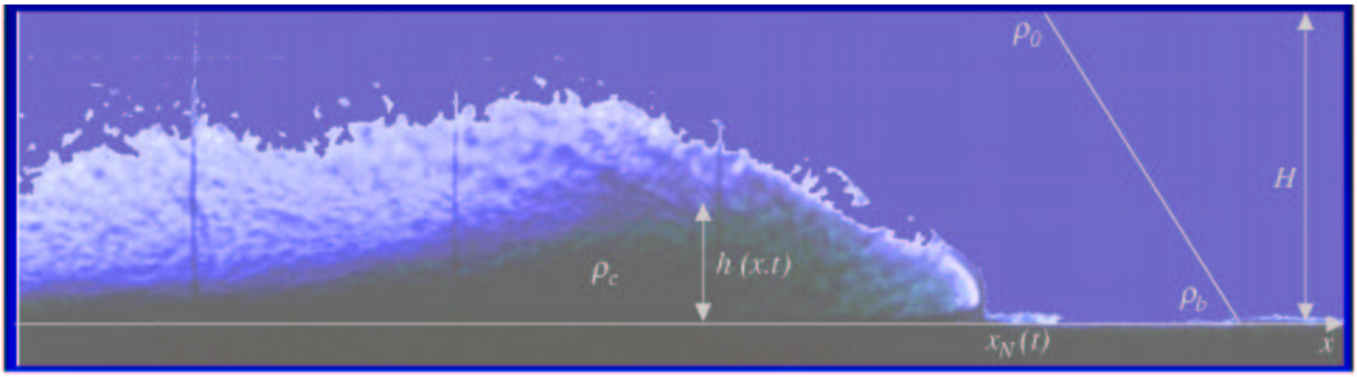

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Introduction/Motivation

Gravity currents (GC) are important phenomena in geophysics and industry. We need understanding and predictive tools.

The body of knowledge for propagation in homogeneous non-stratified ambient is well developed, in particular the shallow-water (SW) formulation.

HOWEVER, in practical applications the ambient is stratified.

Shallow-water (SW) Formulation

The height of the ambient is H. Assume linear decrease of ambient density from ρ_b to ρ_o. Density of current is ρ_c. Define:

$$\epsilon = \frac{\rho_c - \rho_o}{\rho_o}, \quad \epsilon_b = \frac{\rho_b - \rho_o}{\rho_o}, \tag{1}$$

and the "MAGNITUDE" of stratification is

$$S = \epsilon_b / \epsilon \tag{2}$$

Note: $0 \leq S \leq 1$. Now

$$\rho_c = \rho_o(1+\epsilon), \quad \rho_a = \rho_o\left[1 + \epsilon S(1 - \frac{z}{H})\right], \tag{3}$$

The reference reduced gravity,

$$g' = \epsilon g. \tag{4}$$

Assume: large Reynolds number Re (inviscid), and small ϵ (Boussinesq).

2D case recalled briefly

Use $\{x, y, z\}$ system, the current is released from a lock of length x_0, heigh h_0. We use the simplest shallow-water ONE-LAYER model.

Assume: In the ambient the velocity components $u = v = w = 0$ and hence the fluid is in hydrostatic balance and maintains the initial density $\rho_a(z)$. The motion is in the lower layer only, $0 \le x \le x_N(t)$ and $0 \le z \le h(x,t)$.

Hydrostatic: $\partial p_i/\partial z = -\rho_i g$, where $i = a$ or c. We obtain

$$p_a(z,t) = -\rho_o \left[1 + \epsilon\, S(1 - \frac{z}{2H}\right] zg + C, \tag{5}$$

$$p_c(x,z,t) = -\rho_o(1+\epsilon)gz + f(x,t), \tag{6}$$

Pressure continuity on the interface $z = h(x,t)$ determines $f(x,t)$.

Result:

$$p_c(x,z,t) =$$

$$-\rho_o(1+\epsilon)gz + \rho_o g' \left[h(x,t) - S\int_0^{h(x,t)} [1 - z'/H]dz'\right] + C,$$

and consequently

$$\frac{\partial p_c}{\partial x} = \rho_o g' \frac{\partial h}{\partial x}\left[1 - S(1 - h/H)\right]. \tag{7}$$

Next, we perform the z-average of continuity and x-momentum eqs. of dense fluid, using the above $\frac{\partial p_c}{\partial x}$.

A boundary condition for u_N (at the nose) was developed

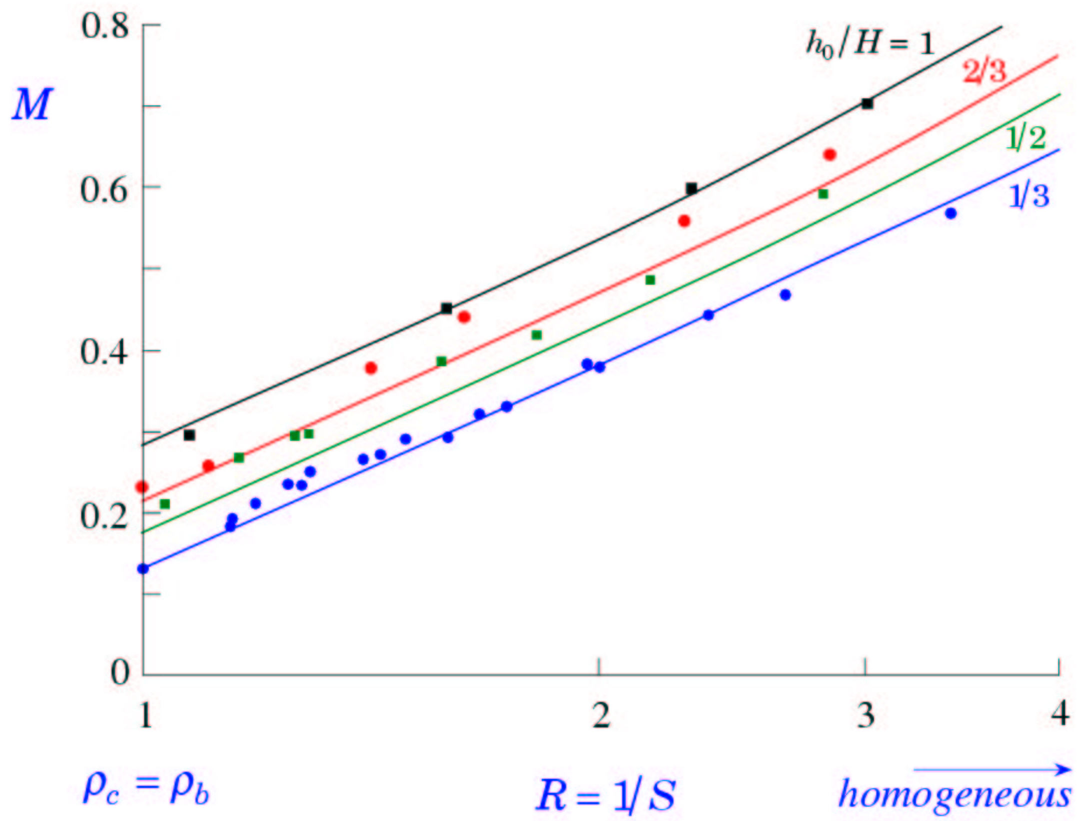
$$u_N = (g'h_0)^{1/2} Fr\, h_N^{1/2} \times \left[1 - S(1 - \frac{1}{2}\frac{h_N}{H}) \right]^{1/2}, \qquad (8)$$

and $Fr = Fr(h_N/H)$, same as in the homogeneous case.

The details of the 2D solution are given in [4].

Compare our 2D results with experiments of [2]

- Qualitatively: initially, u_N is constant and independent of x_0/h_0, in both theory and experiment;
- Quantitatively, we plot our predictions on Fig 7 of that paper and obtain the figure below. (M is the velocity scaled with $(Sg'H)^{1/2}$, points are experiments, line theory).



Good agreement in general ($\pm 5\%$); excellent agreement for $H = 1$. For $H = 2/3$ largest discrepancy, not clear why.

Axisymmetric and rotating cases

The axisymmetric and rotating extensions of the recent 2D investigations are of interest, and we attempt to do it.

The current is released from a cylindrical lock of height h_0 and radius r_0, and the entire system is rotating with a constant angular velocity Ω about the vertical axis z (with $\Omega = 0$ as a particular case).

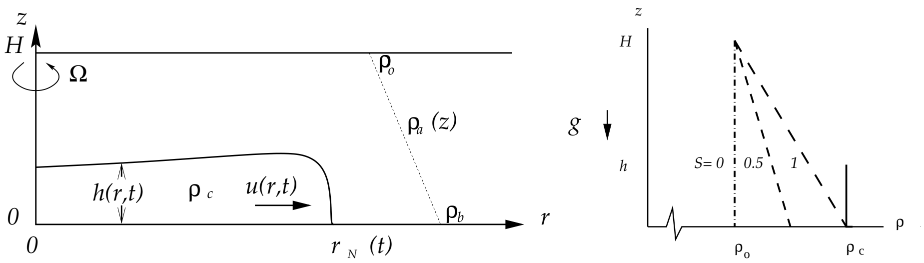
We developed a SW one-layer inviscid theory. We derived governing parameters, qualitative insights and quantitative results. The predicted behaviour is very different from the 2D case.

MOTIVATION and OBJECTIVES of EXPERIMENTS:

We want to verify the theory and gain additional insights. Major points: influence of waves, nose velocity correlation, non-symmetric components, stability.

The Coriolis-Grenoble large tank provides an excellent facility for our purposes. In small containers, observation of details is very difficult and viscous effects are expected to become dominant after a relatively short propagation.

THEORY



The investigation is underway, [5].

Use cylindrical co-ordinates $\{r, \theta, z\}$, rotating with Ω. The velocity components are $\{u, v, w\}$. We assume axisymmetric and inviscid $(Re \to \infty)$ flow. Current released from a cylinder of radius r_0 and height h_0.

Scale the dimensional variables (denoted here by asterisks) as follows

$$\{r^*, z^*, h^*, H^*, t^*, u^*\ v^*\} = \{r_0 r, h_0 z, h_0 h, h_0 H, Tt, Uu, \Omega r_0 v\}, \quad (9)$$

where

$$U = (h_0 g')^{1/2} \quad \text{and} \quad T = r_0/U. \quad (10)$$

We define the angular velocity (in the rotating system)

$$\omega = v/r, \text{ scaled with } \Omega. \quad (11)$$

Dimensionless parameters:

Height ratio $H = H^*/h_0$, lock aspect-ratio h_0/r_0,

stratification $S = (\rho_b - \rho_o)/(\rho_c - \rho_o)$.

Additional dimensionless parameter is the typical Coriolis to inertia ratio

$$\mathcal{C} = \frac{\Omega r_0}{(g' h_0)^{1/2}}. \quad (12)$$

Connected to the familiar frequency ratio

$$\frac{f}{N} = 2\sqrt{\frac{H}{S}}\left(\frac{h_0}{r_0}\right)\mathcal{C}. \tag{13}$$

A hydrostatic-cyclostrophic pressure balance is assumed in the motionless ambient, and a hydrostatic vertical balance in the dense fluid. We find that the relationship between the lateral pressure gradient and inclination of the interface is like in the 2D case.

The internal waves are filtered out. (MUST BE VERIFIED.)

The averaged balance equations of continuity, radial momentum and azimuthal momentum can be written as

$$\begin{bmatrix} h_t \\ u_t \\ v_t \end{bmatrix} + \begin{bmatrix} u & h & 0 \\ 1-S+S\frac{h}{H} & u & 0 \\ 0 & 0 & u \end{bmatrix} \begin{bmatrix} h_r \\ u_r \\ v_r \end{bmatrix} = \begin{bmatrix} -uh/r \\ \mathcal{C}^2 v(2+v/r) \\ -u(2+v/r) \end{bmatrix}. \quad (14)$$

The system is hyperbolic. We solve it by a Lax-Wendroff finite difference method.

The initial conditions are zero velocity in both radial and azimuthal directions, and unit dimensionless height and length at $t = 0$.

Boundary conditions for the radial and angular velocity components at the nose $r = r_N(t)$ are needed.

(1) We argue that for small values of $\mathcal{C}^2$ the boundary conditions for u_N are like in the 2D case. (MUST BE VERIFIED).

(2) Potential vorticity is conserved

$$\frac{D}{Dt}\left(\frac{\zeta+2}{h}\right) = 0, \quad (15)$$

where

$$\zeta = \frac{1}{r}\frac{\partial}{\partial r}(r^2\omega) \quad (16)$$

is the axial vorticity component (scaled with Ω).

A combination of volume continuity and PV conservation yield

$$\omega = -1 + \left(\frac{1}{r_N(t)}\right)^2 \quad (r = r_N). \quad (17)$$

Steady Lenses

For $\mathcal{C} > 0$ the system admits a non-trivial steady-state solution with $u = 0$ and $r_N =$ const. The task is to determine $h(r), \omega(r)$ and r_N of the possible steady lens (SL).

Letting $y = r/r_N$, the the radial momentum and potential vorticity equations for $0 \leq y \leq 1$

$$A\frac{\mathrm{d}h}{\mathrm{d}y} = \mathcal{C}^2 r_N^2 y[\omega(2+\omega)], \tag{18}$$

$$h = 1 + \omega + \frac{1}{2}y\frac{\mathrm{d}\omega}{\mathrm{d}y}, \tag{19}$$

where

$$A = \left[1 - S + S\frac{h}{H}\right] \tag{20}$$

subject to the the boundary conditions (17), regularity at $y = 0$, and $h(y = 1) = 0$. Substitution of (19) into (18) yields a single equation for ω.

For $S = 1$ we obtain $A = h/H$, and a singularity of (18) at $y = 1$ appears due to the conditions $h(1) = 0$.

Analytical approximations for SL

1. $\mathcal{C} \ll 1$ and $1 - S \gg \mathcal{C}$. An expansion in powers of $\mathcal{C}$ indicates that $h \sim \mathcal{C}$, and hence, to leading order, $A = 1 - S =$ Const. Letting

$$\mathcal{C}_m = \mathcal{C}(1 - S)^{-1/2} \tag{21}$$

we obtain the approximation

$$\omega = -1 + \mathcal{C}_m(1 - \frac{1}{2}y^2), \quad h = \mathcal{C}_m(1 - y^2), \tag{22}$$

$$r_N = (2/\mathcal{C}_m)^{1/2}. \tag{23}$$

The stratification decreases the inertial terms and hence increases the relative importance of the Coriolis effects.

2. $S = 1$ and $\mathcal{C}^2 H \ll 1$. Expansion in powers of $(\mathcal{C}^2 H)^{1/3}$ gives

$$h = (\mathcal{C}^2 H)^{1/3} \left(\frac{3}{2}\right)^{1/3} (1 - y^2)^{1/2}, \tag{24}$$

$$\omega = -1 + (\mathcal{C}^2 H)^{2/3} \left(\frac{2}{3}\right)^{2/3} \left[1 - (1 - y^2)^{3/2}\right] \frac{1}{y^2}, \tag{25}$$

$$r_N = \left(\frac{3}{2}\right)^{1/3} (\mathcal{C}^2 H)^{-1/6}. \tag{26}$$

The case $S = 1$ is related to the lenses produced by intrusion in a stratified fluid at a neutral level. The aspect ratio of the dimensional height to the dimensional radius of the lens predicted for $S = 1$ is

$$\Omega \left(\frac{H h_0}{g'}\right)^{1/2} = \frac{1}{2}\frac{f}{N} \tag{27}$$

consistent with previous results for the neutral level intrusion lens from a point source of non-rotating fluid by other authors.

In general, the solution of the SL is performed by numerical methods. The approximate results capture well the solution, see Fig. 1 below. As the stratification (value of S) increases, the lens becomes thicker and shorter and the retrograde angular velocity in the interior decreases.

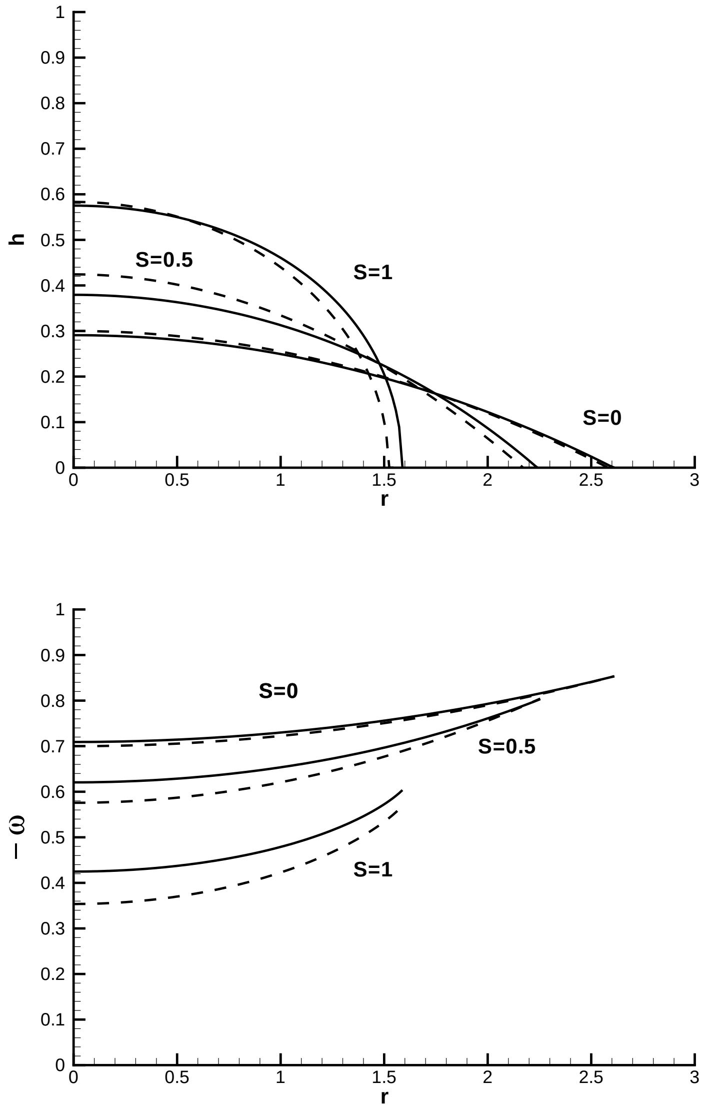


Figure 1: Lens results for $\mathcal{C} = 0.3$, $H = 2$ and various S, numerical solution (solid lines) and approximate solution (dashed lines), for h and ω as functions of r.

SW Results of time-dependent motion

We illustrate in the figure below the effect of stratification on the propagation of a rotating axisymmetric gravity current, as predicted by the SW formulation [5]. The Coriolis effects hinder, and eventually stop, the radial propagation. When S increases both the speed of radial propagation and the maximum radius of spread are significantly reduced. (MUST BE VERIFIED).

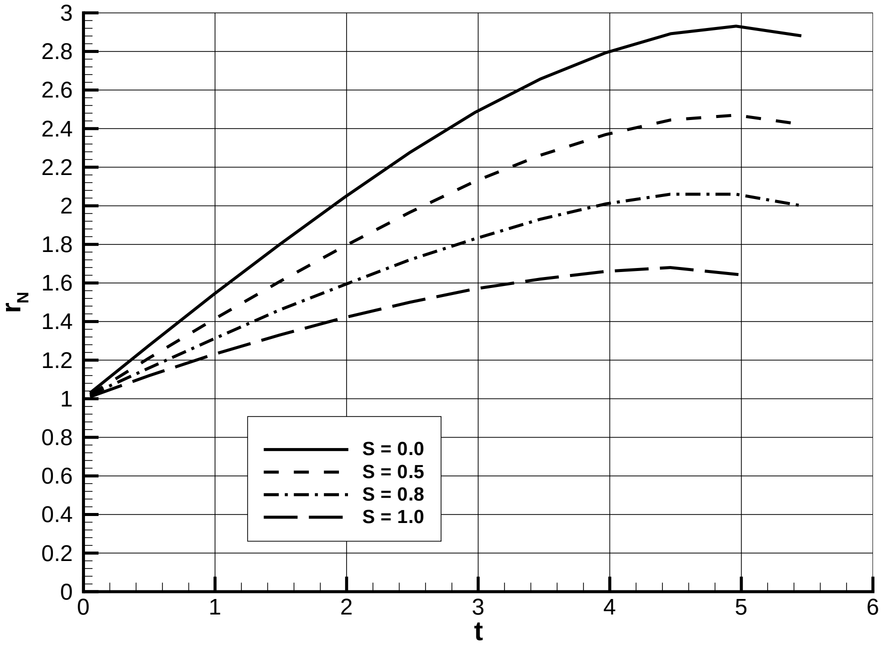


Figure 2: SW results for the propagation of gravity currents rotating with $\mathcal{C} = 0.4$ for various stratification of the ambient. Here $H = 2$.

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EXPERIMENTS in LARGE TANK

- Performed Feb. 17 - March 5, 2003, actually the inauguration of the “new” structure.
- Cylinder lock $r_0 = 100\mathrm{cm}$ radius, typical $h_0 = 30\mathrm{cm}$.
- Ambient with various stratifications, $0 \leq S \leq 1$, typical height $80\mathrm{cm}$.
- $\Omega = 0.1$ and $0.05\,\mathrm{s}^{-1}$.
- The ambient was spun-up, and then the small lock cylinder was filled with dense ρ_c fluid, while the ambient from inside is removed at the same rate (tricky, but it worked).
- The dense fluid (current) was marked by fluorescein and particles were mixed in.
- Density profiles were recorded by probes on computer files.
- Experiment “run” start: lift the lock (by motor) in about 2 s above the ambient, see Fig. 3 below.
- Motion was marked by laser and recorded on computer. Propagation and velocity field obtained via the PIV computer software of Coriolis Laboratory.
- After the run, the dense fluid was let to settle, and a $2\mathrm{cm}$ layer was removed from the bottom. Then, a new “run” was prepared.

- Typical conditions: $\rho_b = 1.02$, $\rho_c = 1.03\,\mathrm{gr/cm}^3$, $\Omega = 0.1\,\mathrm{s}^{-1}$
  $S = 0.6$, $\mathcal{C} = 0.3$, $H = 2.5$, $h_0/r_0 = 0.3$,
  $U = 30\mathrm{cm/\,s}$, $N = 0.5\,\mathrm{s}^{-1}$, $N/f = 2.5$, $r_{Nmax} = 250\mathrm{cm}$. $Re \sim 10^4$

- We had some difficulties to get started with the first experiments, and with the computer code. We performed seven "good" runs that are still under analysis. Some results are shown in Fig. 4 below.

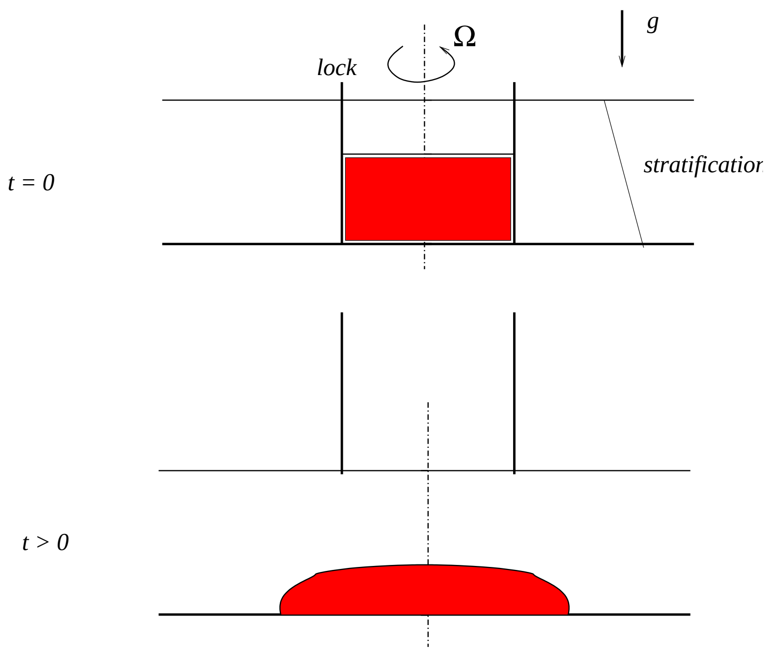

Figure 3: Schematic description of release of current (red) in experiment.

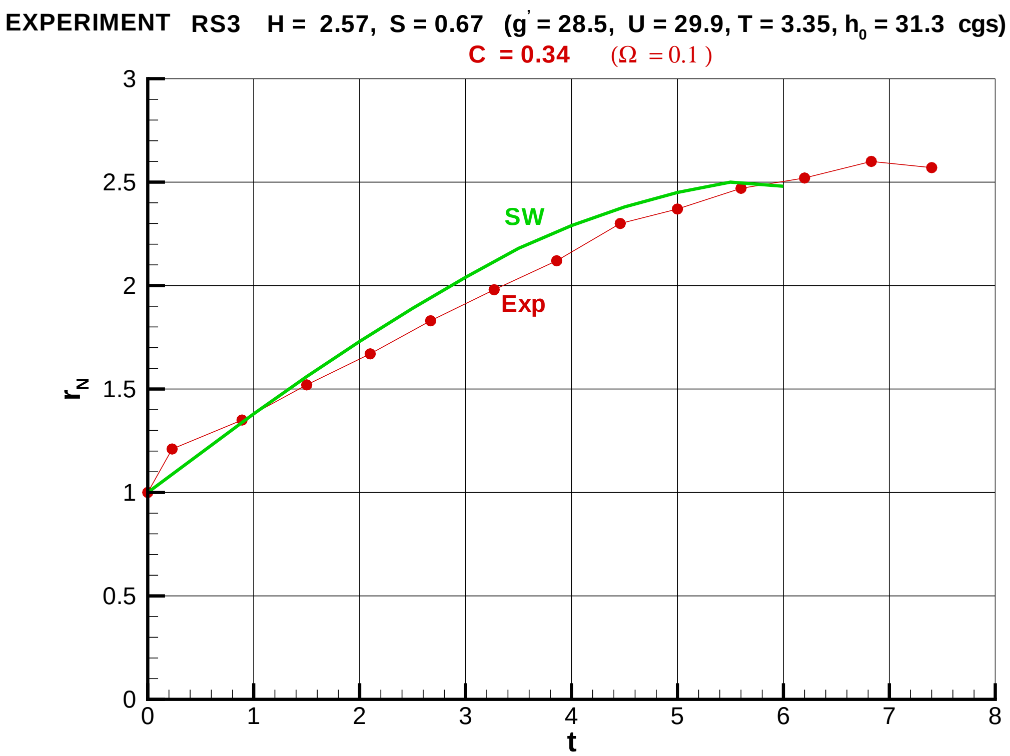

Figure 4: Radius of propagation as a function of time, experimental points and SW theory prediction for one typical experiment.

Acknowledgments

The research was supported by NERC, EPSRC, and by the Technion V.P.R. Fund - New-York Metropolitan Research Fund.

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